ECO 799: THESIS



United International University

An Empirical Multivariate Time Series Analysis of the United States’ Production Function

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Program: Master’s of Science

In Economics

**Submission Date**

25th September, 2018

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An Empirical Multivariate Time Series Analysis of the United States’ Production Function

# Abstract

This paper attempts to empirically review the United States’ production function.The impact of **hours worked/capital** that is average weekly hours of production divided by capital stock, annual government total expenditures divided by capital stock as **government expenditure/capital**, **inflation rate** and **relative price of gasoline** as proxy for energy on**output/capital** which is GDP divided by capital stock in United States. 40 years of annual data from 1975 to 2015 is collected for each variable. Co-integration and VECM model with several diagnostic tests has been conducted in this study. However, only hours worked/capital and relative price of gasoline were significant at the beginning but inflation rate and governmentexpenditure/capital were also made significant after imposing normalized restriction through co-integration.

IntroductionPreviously so many studies were conducted on production function. In most of the papers the main focus was on estimating the relative or absolute relationship between output and inputs as, capital, labour and technical progress. The labour mostly was measured by total hours of work by the labours, capital was measured with private capital stock and technology was measured on the basis of time trend. The production function measurement procedure has changed over time. In the earlier period the production function estimations were done through estimating transformed model which shows relationship among the dependent and independent variables but now production function is measured in many parts. The impact on output of inflation, price of energy and private capital has specific hypothesis which was upgraded by the production function researchers.

United States is a country that generates the highest nominal GDP and it is also known to be under possession of the second highest position in Purchasing Power Parity. United States earns 80% of their total GDP form the service sector and approximately 19% from Industrial sector (Central Intelligence Agency World Fact Book, Central Intelligence Agency. Retrieved April 3, 2018). Currently this country holds $20.412 trillion dollars of GDP in 2018 (IMF, Retrieved April 3, 2018.)

In United States the work limit for a labour is 8 hours a non-school day and 40 hours in a non-school week. Number of full-time employees in United States till 2017 was 125.97 million people and they earn a total of 14.96 USD per hour on average (www.statista.com).

Capital stock represents the number of issued shares of a company by the shareholder. The private capital stock seems to have both direct and indirect influence from public sector. Intermediate service to private sector is the reason behind direct influence and correlativity between the private and government capital is an indirect cause. In earlier studies, it has been seen that, local capital and state have an impact on private capital output which is significantly different from zero with a positive value but in recent times insignificance has taken over. As United States is a capital intensive country which means the production procedure is less dependent on labour force the relationship between the capitals should be significant.

The economic agents suffer from inflation for wasting time on purpose of money circulation. Inflation rate of United States currently falls to 2.7% in late August 2018, previously which was 2.9% on July 2018 ([www.trandingeconomics.com](http://www.trandingeconomics.com)) which is due to the mild reduction in fuel price. On average from 1914 to 2018, inflation rate is 3.27 with highest recorded 23.7% and lowest -15.8 in the year 1920 and 1921 respectively. It is considered that, inflation has a deep-rooted impact on output and relationship between them should bring out an outcome of less consumption.

The ratio of output over price of energy noticeably varied after the 80s it increased from 1973 to 1985 (Energy Information Administration, Monthly Energy Review, Table 9.4.) and decreased after then.

The focus of this study is to see the consequences of plotting; total hour worked to private capital ratio, relative price of energy, ratio of government expenditure on private capital and inflation on ratio of GDP to private capital of United States. In earlier studies the single equation models were used to estimate cobb-doglus production function, but alike recent studies in this paper to analyse, Vector Auto-Regressive (VAR) model is used.

# Limitations of the Study

Data manipulation was done before generating the results, as all raw data sets were not available annually and presumed form.

# Literature Review

In this section the previous studies on production function will be discussed elaborately. There are so many papers conducted on production function by taking different types of variables and measurement procedure. In this paper, studies with Vector Auto Regressive (VAR) model will be given priority.

Shaiara Husain & Md. Shahidul Islam (2016) conducted a paper named ‘A Test for the Cobb Douglas Production Function in Manufacturing Sector: The Case of Bangladesh’. Here the production function contained one dependent and two independent variables named total value of the output, total liabilities and number of permanent workers. A natural log transformed linear in the parameters (log-linear) regression model is used to explain the study. Diagnostic tests like DW&BPG are tested to comment on the violation of CLRM assumption. Constant Returns to Scale is checked with the help of Restricted Least Square model. The result section suggests that, the Bangladesh Industry tends to have constant returns to scale for the production function but after adjusting for the unit root tests, it showed increasing returns to scale.

[2] ‘A Multivariate Co-integration Analysis of the Role of Energy in the US Macro-economy’ is also conducted with a VAR model. It was conducted by David I. Stern (2000). Here, in the initial stage the VAR model was constructed and then it was also tested with co-integration and Vector Error Correction Model with an annual data set from 1950 to 1973. This data set was taken of the dependent variable GDP and independent variables capita, energy & labour. The result shows that there is co-integration relationship among GDP, capital, labour and energy which shows a long-run effect. This paper supports Granger causality between energy and GDP and energy is a limiting factor as reduce in energy supply will lead to lower GDP output.

[3]Vector Auto Regressive model was conducted by W. Douglas Mcmillin& David J.Smyth (1994) on a paper ‘A Multivariate Time Series Analysis of the United States Aggregate Production Function’ that includes ratio of hours worked and private capital, relative price of energy, inflation and ratio of government capital and private capital as independent variables and ratio of private sector output and private capital is a dependent variable. Moving Average is used on the basis of time series data set from 1952 to 1990. The paper showed that the independent variables have significant impact on the dependent variable rather than the government capital per unit of private capital. This paper concludes on a belief that, it is necessary to increase the stock of government capital to stimulate productivity in the private sector.

[4]With a dependent variable value of output (in Rupees) and independent variables total area of land, total number of hours of human labour, total number of hours of bullock labour and total value of input costs, J.E. Battese& T.J. Coelli (1992) introduced a paper ‘Frontier Production Functions, Technical Efficiency and Panel Data: With Application to Paddy Farmers in India’. Data of 38 Indian agricultural firms is used to create stochastic frontier production function that is time invariant. A log transformed liner model is estimated which is linear in the parameters and the time-varying model. The Maximum Likelihood Estimates are included for the parameters. Mean values of paddy from 1975 to 1985 were forecasted to support the study. Results suggested that, almost all the independent variables have significant relationship with the dependent variable. And technical inefficiency is not an issue for the country which means it can be mitigated by adding additional labours to produce output.

[5]Melvyn A. FUSS (1977) conducted a thesis on ‘The Demand for Energy in Canadian Manufacturing an Example of the Estimation of Production Structures with Many Inputs’ that includes gross output as dependent variable and energy, labour, materials and capital as independent variables. A natural log transformed linear in the parameters (log-linear) regression model that is a non-homothetic production structure is used for the paper. It contains squared variables. A two stage model is also explained in this study. In result section it can be seen, Three evident are conclusive in this paper, one is substantial inter-fuel substitution is possible, another is only moderate substitutes of energy for other aggregates and the last one large increases in energy prices can be accommodated with only small output price increases.

Taking log values of Los Angeles wage rate as dependent variable; and national wage rate as independent variable, [6]Robert Engle & Mark Watson (2012) instigated a paper named ‘A One-Factor Multivariate Time Series Model of Metropolitan Wage Rates’. This paper is estimated by maximum likelihood methods using the Kalman filter algorithm. The model is used to obtain estimates of the unobserved metropolitan wage rate for Los Angeles Hypothesis tests, model diagnostics, and out-of-sample forecasts, and factor analysis are used to evaluate the model. This paper has two different factor analysis models (A & B). Model A wasn’t much satisfying for being misspecified so the model B was introduced and it showed highly significant values for the factors based on the standard error values. Here the metropolitan wage rate varies from its normal relation with the national average after adjusting for industrial mix.

In 2004 [7]Khalifa H. Ghali and M.I.T. El-Sakka lunched a paper on ‘Energy use and output growth in Canada: a multivariate co-integration analysis’. This paper consist of a dependent variable Aggregate output or real GDP and three independent variables capital stock, total employment, total energy consumption. The time series data was collected from 1961- 1997.This analysis strongly suggests that, energy can be considered as a limiting factor to output growth in Canada. And a shock to energy would cause a 15% change in the future growth rates of output.

One of the most unique papers discussed in current thesis is a case that was constructed with the help of primary data. Donald S. Kenkel[8] in 1995 presented a question based paper ‘Should You Eat Breakfast? Estimates from Health Production Function’. A health production function approach is used for this paper, here the importers of health in adulthood lifestyle is evaluated with the practice of eating breakfast, smoking, and exercise. This paper includes primary data collection method. The dependent variable here is health outcomes, that asks health status, activity limitation, blood pressure, weight to male and female both and the independent variables are health inputs that includes askings related to breakfast, snacking, smoking, exercise, drinking, sleep, stress; Chronic conditions includes heart trouble, diabetes, stroke; Socio-economic includes race, age, schooling, hispanic. Though the paper has some doubts on the interpretation for the lacking of support it concludes that  
practices such as smoking, drinking, and exercising, but not eating breakfast founded to be related to different health issues. And the second objective schooling, found to be related to good health. This paper showed that lifestyle has a strong connection with health.

[9]Muhammad Shahbaz, Saleheen Khan & Mohammad IqbalTahir (2013) introduced "The dynamic links between energy consumption, economic growth, financial development and trade in China: Fresh evidence from multivariate framework analysis" with dependent variable real GDP and independent variables energy use, real domestic credit to private sector as a proxy for financial development, real trade openness and real capital use. This paper investigated the relationship of energy and economic growth by incorporating financial development, international trade and capital under a production function. The ARDL bounds testing approach to co-integration was applied under stationary properties by applying structural break test to show the long-term relationship among the variables. China is becoming more and more energy efficient over time. In China, increasing energy use will lead to real GDP increase. But the growth of energy use does not have direct one to one correlation with GDP growth. The Chinese government should make every effort to improve energy efficiency and reduce pollutant emissions.

[10]UgurSoytas&Ramazan Sari (2006) constructed ‘Energy consumption and income in G-7 countries. Here, G-7 represents Canada, France, Germany, Italy, Japan, United Kingdom, and United States. This paper was initiated with Vector Error Correction Model (VECM) with five different unit-root tests were performed to check time series properties for this particular test. Johansen co-integration test & generalized variance decomposition was also employed. Real GDP per capita as dependent variable and total labour force and total energy is used as independent variables. This paper provided policy implication suggestions for the G-7 countries separately. For UK, Canada Italy and Japan balanced combination of alternative policies looks appropriate. United States and France can focus on technological development and mitigation policies. And Germany should focus on the global warming issue through energy conservation measures.

[11]David I. Stern’s another paper published in 1993 ‘Energy and economic growth in the USA’. Here, the VAR model was constructed with four equations with a constant dollar of US GDP. The capital is product of stock capital and capacity utilization. GDP output is used as a dependent variable and labour, capital & energy are used as independent variables. The results suggested that, raising taxes on energy or adopting other policies that cut energy use without specifying the ways in which energy use should be reduced.

# Methodologyand Result Discussion

This analysis was done with a secondary data set of United States’ stock market capitalization to GDP[13],capacity utilization[14], retail gasoline price (constant dollars 2015/gallon)[15], Federal government total expenditures in billions[16], inflation rate[17], realgross domestic product[12] in billions, and average weekly hours of production[18]. Datawere taken from ‘Federal Reserve Bank of St. Louis’, ‘Energy Information Administration’ and ‘[Kimberly Amadeo’s](https://www.thebalance.com/kimberly-amadeo-3305455)forecasting inflation year by year’ for in-totalof 40 years annually from 1975 to 2015.

The product of stock market capitalization to GDP and capacity utilization represents the total capital for United States. Inflation rate was taken as decimal form. Many of the variables were not found annually so at first the summed form was taken of the monthly and quarterly data sets.

As this Paper includes production function, the measurements of ratio scales were taken for some variables. The dependent variable here is the **output/capital** which is realgross domestic product divided by product of stock market capitalization to GDP and capacity utilization represents the total capital. Independent variables are **hours/capital** that is average weekly hours of production divided by product of stock market capitalization to GDP and capacity utilization represents the total capital, annual government total expendituresdivided by product of stock market capitalization to GDP and capacity utilization represents the total capitalas **government expenditure/capital**, **inflationrate**and takingrelative price of gasoline price asProxy for **relative price ofenergy.**

Statistical software STATA was used for analysing an econometric model through some tests. The model is specified in the bellow part which shows the association among the dependent and independent variables.

For this particular model the Ordinary Least Square (OLS) regression model is considered to be a weak statistical test, and based on the variables and data it will be suitable to use Vector Error Correction Model (VECM).The Vector Error Correction Model (VECM) with Co-integration analysis is taken with the help of generating unit-root Augmented Dickey-Fuller test, Prais-Winsten, Newey-West, Granger Causality test, Diagnostic tests, White Noise error term, Lag-range Multiplayer test.

Now forconductingVECM, the unit-root test Augmented Dickey-Fuller (ADF) test for each variable were done which shows all the variables were non-stationary in the level form and 1st differential form but stationary in the 2nd differential form. The results for level, 1st differential and 2nd differential are provided in table-1.

**Augmented Dickey-Fuller test for unit root**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Table-1: Augmented Dickey-Fuller test | | | | | | | **Integrated Level** |
| **Order** | **Level Form** | | **1st Differentials** | | **2nd Differentials** | |
| **Variables** | **Cal. Value** | **95% C.I. (P-value)** | **Cal. Value** | **95% C.I. (P-value)** | **Cal. Value** | **95% C.I. (P-value)** |
| **Output/Capital** | -1.282 | -1.701  (0.1052) | -1.798 | -1.703  (0.0417)\* | -3.246 | -1.706  (0.0016)\* | **I(2)** |
| **Hours Worked/Capital** | -1.263 | -1.701  (0.1084) | -1.786 | -1.703  (0.0427)\* | -3.614 | -1.706  (0.0006)\* | **I(2)** |
| **Relative Price of Energy** | -1.651 | -1.701  (0.0550) | -1.427 | -1.703  (0.0826) | -2.307 | -1.706  (0.0146)\* | **I(2)** |
| **Govt. Expenditure/Capital** | -1.029 | -1.701  (0.1561) | -1.974 | -1.703  (0.0293)\* | -2.826 | -1.706  (0.0045)\* | **I(2)** |
| **Inflation rate** | -4.334 | -1.701  (0.0001)\* | -3.466 | -1.703  (0.0009)\* | -5.264 | -1.706  (0.0000)\* | **I(2)** |

For that the Auto-Regressive Integrated Moving Average (ARIMA) model (I (2)) must be performed.

**Prais-WinstenAR (1) regression**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Tabel-2:Prais-Winsten | | | | | | | |
| **D2\_Output/Capital** | **Coefficient** | | **Std. Err.** | | **t-value** | **P>|t|** | |
| **Constant** | -.0012161 | | .0055307 | | -0.22 | 0.827 | |
| **D2\_ Hours Worked/Capital** | 151.725 | | 10.01878 | | 15.14 | 0.000 | |
| **D2\_ Relative Price of Energy** | .5112693 | | .0413158 | | 12.37 | 0.000 | |
| **D2\_ Govt. Expenditure/Capital** | .0110494 | | .0137316 | | 0.80 | 0.427 | |
| **D2\_ Inflation rate** | .1440214 | | .2212131 | | 0.65 | 0.519 | |
| **rho(**ρ**)** | -.0732101 | | | | | | |
| F( 4, 34) = 402.05 | | Prob> F = 0.0000 | | R-squared = 0.9793 | | | n = 39 |

Durbin-Watson statistic (original) 2.083231

Durbin-Watson statistic (transformed) 1.999242

Generalized Least Square (GLS) method is used to test Prais-Winsten test to take care of the serial correlation if present in the model.Dependent variable and independent variables are transformed into 2nd differential form. It is because of (I (2)) generated from the ADF unit-root test. The lag length or selection order was taken as *l*= 5, holding max lag order as 5 LR, FPE, AIC, HQICand SBIC suggested lag length to be 5.

The results of the Prais-Winsten test in table-2 shows that, dependent variable Output/Capital is positively related to the independent variables so; increase of a unit in independent variable will lead to a unit change in dependent variable. By evaluating the p-value it can be said that, Hours Worked/Capital & Relative Price of Energy are significantly different from zero but other two variables are insignificant and the constant is also insignificant which means it is a regression trough the origin. For the overall significance, R-squared shows that 97.93% of output/capital can be explained by the independent variables and F-value 402.05 with df (4, 34) and p value 0.000 is also significant. Though two variables are not significant butt overall model is significant.

Durbin-Watson statistic is approximately 2 so it can be said that there is sign of auto-correlation.

**Regression with Newey-West standard errors**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Table-3: Newey-West Test | | | | | | | |
| **D2\_Output/Capital** | **Coefficient** | | **Std. Err.** | | **t-value** | **P>|t|** | |
| **Constant** | -.0011651 | | .005958 | | -0.20 | 0.846 | |
| **D2\_ Hours Worked/Capital** | 151.528 | | 12.21519 | | 12.40 | 0.000 | |
| **D2\_ Relative Price of Energy** | .5123744 | | .0361033 | | 14.19 | 0.000 | |
| **D2\_ Govt. Expenditure/Capital** | .0101743 | | .0082199 | | 1.24 | 0.224 | |
| **D2\_ Inflation rate** | .1178003 | | .29741 | | 0.40 | 0.695 | |
| F( 4, 34) = 340.04 | | Prob> F = 0.0000 | | Max Lag = 0 | | | n = 39 |

Newey-West test is used in a model if auto-correlation is detected in earlier studies. Though in early segment no auto-correlation was detected, still it is used to compare the results with Prais-Winsten test. The results of the Newey-West test show almost the same results as the previous one. As the number of observations and dependent variable is same it is comparable.

**Vector Error-Correction Model**

**Johansen tests for co-integration**

Trend: trend Number of obs = 39

Sample: 1977 - 2015 Lags = 2

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Maximumrank** | **parms** | **eigenvalue** | **Tracestatistic** | **5%cri. value** |
| 0 | 35 | - | 88.2968 | 77.74 |
| 1 | 44 | 0.62038 | **50.5224\*** | 54.64 |
| 2 | 51 | 0.38804 | 31.3700 | 34.55 |
| 3 | 56 | 0.33308 | 15.5719 | 18.17 |
| 4 | 59 | 0.22752 | 5.5041 | 3.74 |
| 5 | 60 | 0.13162 | - | - |

## Table-4: Johansen tests for co-integration

Johansen tests for co-integration shows the rank at which the model is co-integrated. Table-5 shows that the, maximum eigenvalue and the trace statistics, for all variables there exists at-least one co-integrating equation. The equation is given in Table-6 that also showed the high significance level of the equation. If there is at-least one co-integration for the variables, existence of Granger causality exists but it doesn’t specify the direction. To do so it is important to generate Vector error Correction Model (VECM) rather than Vector Auto-Regressive Model (VAR). The Granger Causality test is performed for the Vector Error Correction Model.

**Co-integrating equations**

|  |  |  |  |
| --- | --- | --- | --- |
| Table-5: Co-integrating Equations | | | |
| **Equation** | **Parms** | **chi2** | **P>chi2** |
| \_ce1 | 4 | 186.736 | 0.0000 |

Identification: beta is exactly identified

**Diagnostic tests of the estimated VEC model**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Variables | **Output/Capital** | **Hours Worked/Capital** | **Relative Price of Energy** | **Govt. Expenditure/Capital** | **Inflation rate** |
| **R2** | 0.0726 | 0.1296 | 0.4234 | 0.1623 | 0.2769 |
| **Jarque and Bera** | 0.46602 | 0.33924 | 0.01533 | 0.98223 | 0.10871 |

## Table-6: Diagnostic tests of the estimated VEC model

The overall JB test shows that the residuals are normally distributed.

**Granger causality Wald tests**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table-7: Granger Causality Wald Tests | | | | | |
| **Equation** | **Excluded** | **F** | **df** | **df\_r** | **Prob> F** |
| **Output/Capital** | Hours Worked/Capital | 3.8637 | 5 | 10 | 0.0328\* |
| Govt. Expenditure/Capital | 3.9654 | 5 | 10 | 0.0304\* |
| Inflation rate | 3.381 | 5 | 10 | 0.0478\* |
| Relative Price of Energy | 5.7728 | 5 | 10 | 0.0092\* |
| ALL | 3.4111 | 20 | 10 | 0.0252\* |
| **Hours Worked/Capital** | Output/Capital | 5.654 | 5 | 10 | 0.0099\* |
| Govt. Expenditure/Capital | 6.5175 | 5 | 10 | 0.0060\* |
| Inflation rate | 5.8593 | 5 | 10 | 0.0088\* |
| Relative Price of Energy | 9.2365 | 5 | 10 | 0.0016\* |
| ALL | 6.156 | 20 | 10 | 0.0027\* |
| **Govt. Expenditure/Capital** | Output/Capital | 2.5795 | 5 | 10 | 0.0949 |
| Hours Worked/Capital | 2.9809 | 5 | 10 | 0.0666 |
| Inflation rate | 2.4806 | 5 | 10 | 0.1038 |
| Relative Price of Energy | 2.3087 | 5 | 10 | 0.1219 |
| ALL | 2.0636 | 20 | 10 | 0.1194 |
| **Inflation rate** | Output/Capital | 3.8687 | 5 | 10 | 0.0327\* |
| Hours Worked/Capital | 6.5149 | 5 | 10 | 0.0061\* |
| Govt. Expenditure/Capital | 1.5552 | 5 | 10 | 0.2580 |
| Relative Price of Energy | 2.0117 | 5 | 10 | 0.1623 |
| ALL | 3.1176 | 20 | 10 | 0.0342\* |
| **Relative Price of Energy** | Output/Capital | 1.6884 | 5 | 10 | 0.2247 |
| Hours Worked/Capital | 1.9839 | 5 | 10 | 0.1668 |
| Govt. Expenditure/Capital | 0.98107 | 5 | 10 | 0.4746 |
| Inflation rate | 1.2306 | 5 | 10 | 0.3636 |
| ALL | 0.97946 | 20 | 10 | 0.5393 |

A certain variable depend on another variable that doesn’t necessarily implies causation. The Granger Causality test shows causation. In this test the null hypothesis is H0: Lagged Hours Worked/Capital doesn’t cause Output/Capital; and alternative hypothesis is Ha: Lagged Hours Worked/Capital causes Output/Capital. As the P-value is greater than the F-value, H0is rejected. So lagged Hours Worked/Capital causes Output/Capital. Prob> F values suggested that, equation of stared values are caused by the excluded variables at 95% C.I.

**Johansen normalization restriction imposed**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **beta** | **Coef.** | **Std. Err.** | **z** | **P>|z|** |  |
| **\_ce1** |
| **\_cons** | 5.486952 | - | - | - |
| **Output/Capital** | 1 | - | - | - |
| **Hours Worked/Capital** | -158.0844 | 23.57302 | -6.71 | 0.000 |
| **Relative Price of Energy** | -.609057 | .1401575 | -4.35 | 0.000 |
| **Govt. Expenditure/Capital** | -.0730919 | .0376665 | -1.94 | 0.052 |
| **Inflation rate** | 12.33556 | 1.99148 | 6.19 | 0.000 |

## Table-8: Johansen normalization restriction imposed

In the case where there is a single co-integrating relation it is possible to identify the vector by a single restriction and often this is the imposition of a single unit coefficient. By setting the vector to zero it is possible to reformulate the co-integrating vector as a long-run equation and in this case the coefficients in the vector are the reverse of the equation normalised in this way. An alternative is to normalise on -1 and then the resulting coefficients in the co-integrating vector are the same as those in the long-run equation. For that reason the variables’ coefficients in earlier were all positive and in this restricted the coefficients are negative. In this restricted model it can be said that all the variables are significant though Govt. Expenditure/Capital is insignificant for the p-values but it’s quite possible to stat that it is near significant.

**Lagrange-Multiplier test**

|  |  |  |  |
| --- | --- | --- | --- |
| Table-9: Lagrange-Multiplier Test | | | |
| **lag** | **chi2** | **df** | **Prob> chi2** |
| **1** | 32.4454 | 25 | 0.14553 |
| **2** | 24.4159 | 25 | 0.49546 |

H0: no autocorrelation at lag order

Evaluating P-values results that, H0 cannot be rejected so there is no auto-correlation.

**Portmanteau test for white noise**

|  |  |
| --- | --- |
| Table- 10: Portmanteau test for white noise | |
| Portmanteau (Q) statistic | 28.6569 |
| Prob> chi2 (17) | 0.0378 |

As the P-value is less than 5%, the null hypothesis H0: Variable follows white noise process can be rejected. So there is no existence of white noise process.

# Findings and Policy Implications

The multivariate time series analysis shows causal relationship of output/capital by hours/capital, inflation rate and govt. expenditure/capital. There is no auto-correlation present in this study, no white noise error term was detected and almost all the variables are normally distributed. The Naway and Prise tests show the coefficients of hours worked/capital and relative price of energy is significant but government expenditure/capital and inflation rate isn’t. Appointing co-integration method helped the model to impose restriction for rank-1 and making all the beta values significantly different from zero in the long-run.

Prediction can be made about the biggest economy in the world that itmay face real GDP to be unaffected by government expenditure and inflation rate in the short run but will become relevant in the long run process. So U.S. government can spend on private sector that will eventually benefit the government as the positive outcome should wait in the long-run.

The inflation rate may not affect the real GDP at short-run period but in the long-run the inflation rate may increase at a level where it can affect the real GDP. So the U.S. government can focus more on service sector to increase Real GDP globally, as the lion’s share of their gross domestic product comes from service sector.

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